Investigation of the Fourier Series

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Abstract

A Graphical User Interface (GUI) was created to demonstrate the Fourier series in a clear and concise manner. It has a simple layout with self explanatory options and labels and explanations where needed. The sum of the Fourier series to an n^{th} term for the approximations of square, sawtooth and triangle waves are calculated within the program. They can be plotted in a canvas widget for easy visual comparison and understanding. Animations of the series with increasing values of n are available too. The user can witness the Gibbs phenomenon [1] which is apparent in these graphs.

1 Introduction

Using the Tkinter package within Python, a GUI was created to demonstrate the Fourier series and its properties. The aim was to make the Fourier series accessible to the user and for the demonstrations to aid comprehension. This was acheived through keeping to a simple layout and by ensuring the user can understand the connections between what they are experiencing and the outputs on the graphs. This report will cover the theory used in the GUI, then demonstrate it's application within the code.

2 The Theory

Any periodic function $f(x)$ can be represented as an approximation in terms of a collection of orthogonal functions. The Fourier series is where periodic functions can be written as a linear combination of sines and cosines (or in the form of an exponential). This has physical applications such as vibrations, oscillations and waves, for example the result of plucking a guitar string. Equation 1 is the approximation with a period L.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right] \tag{1}$$

where a_0, a_n and b_n are constants, n is the n^{th} term, x is the indepedent variable and L is the period of the function.

2.1 Convergence of a Fourier Series and the Gibbs Phenomenon

The Fourier series representation of the function $f(x)$ tends towards half way between the jump and the position of the jump. At a discontinuity there is a visible *overshoot*, called the Gibbs phenomenon [1]. Interestingly, the addition of more terms makes the overshoot tend towards a limit, but never disappear. What does change however is the distance from the oscillations to the jump disconuities. The overshoot gets closer and closer to the discontinuity and the area inside it tends to zero [3] which can give the appearance of it vanishing with $n \to \infty$ (Section 3.2).

3 The GUI

The GUI created has the intention of allowing the user to visually demonstrate, and understand the effects of, increasing the number of terms to which a Fourier series is summed to.

Using Equation 1 and the corresponding coefficient calculations [2], simplified equations for each type of Fourier series were created. The equation and code used to calculate the sum of the values up the the n^{th} term of a square Fourier series is as follows

```
self.Na = int(self.N.get());
L = ma.pi
fourier=[]
var=np.linspace(0,2*L,1000)
f=0
for a in range(0,1000):
        for n in range(1,self.Na,2):
        b=4/(ma.pi*n)
                f+=(b*ma.sin((n*var[a]*ma.pi)/(ma.pi)))
        fourier.append(f)
        f=0
```

With slight differences in the sawtooth calculation,

```
var=np.linspace(0,4*L,2000)
f=ma.pi/2
for a in range(0,2000):
        for n in range(1,self.Na):
                b=ma.pi*-1./(n*ma.pi)
```

and the triangle calculation,

```
var=np.linspace(0,4*L,2000)
f=0
for i in range(0,2000):
        for n in range(1,self.Na,2):
                a=8./((n**2)*(ma.pi**2))
                f+=(a*ma.cos(n*var[i]*ma.pi/L))
```

TALK ABOUT THE DIFFERENCES

3.1 The Master Window

The initial, or master, window includes 5 buttons, as displayed in Figure 1. These buttons are made to be self explanatory to the user, but there is also the option of a 'Help' button available to open an information window with further explanation.

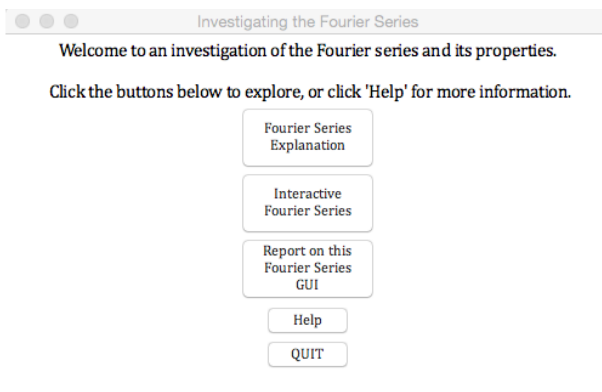

Figure 1: Screenshot displaying the master window including a text widget and 5 button widgets..

3.2 The Central Component

The user is able to input integer values into a tkinter entry widget, and plot a chosen type of Fourier series approximation onto a subplot, as displayed in Figure 2. The subplots allow easy visual comparison between the types of Fourier series.

Figure 3 displays the approximation, to the millionth term, of the three types of Fourier series investigated in the GUI. The user has access to this graph through the use of the 'One Million' button. The reason that the Gibbs phenomenon [1] does not appear in this figure is due to the precision of the graphs. The run time of graphs with greater precision is very large but if calculated, the overshoot would be shown to not have vanished but be tending towards the same limit.

Finally, apart from the option to quit, the last available function is the option to animate the three Fourier Series in increasing values of n. By choosing from the dropdown list and clicking 'Animate', a new window will open with the animation. This enables the user to witness directly the Gibbs phenomenon tending towards it's finite limit, and the smoothing out of the functions.

References

[1] Eric Weisstein, Gibbs Phenomenon, Dec 1, 2014 http://mathworld.wolfram.com/GibbsPhenomenon.html

[2] R. D. Stuart, Fourier Analysis, Spottiswoode Ballantyne and Co Ltd, 1961, pp. 11-13

[3] Ricardo Radaelli-SanchezRichard Baraniuk, Gibbs Phenomena. OpenStax CNX. Dec 9, 2014 http://cnx.org/contents/219c40e2-9b36-4529-ad00-75b744150e85@15/Gibbs_Phenomena

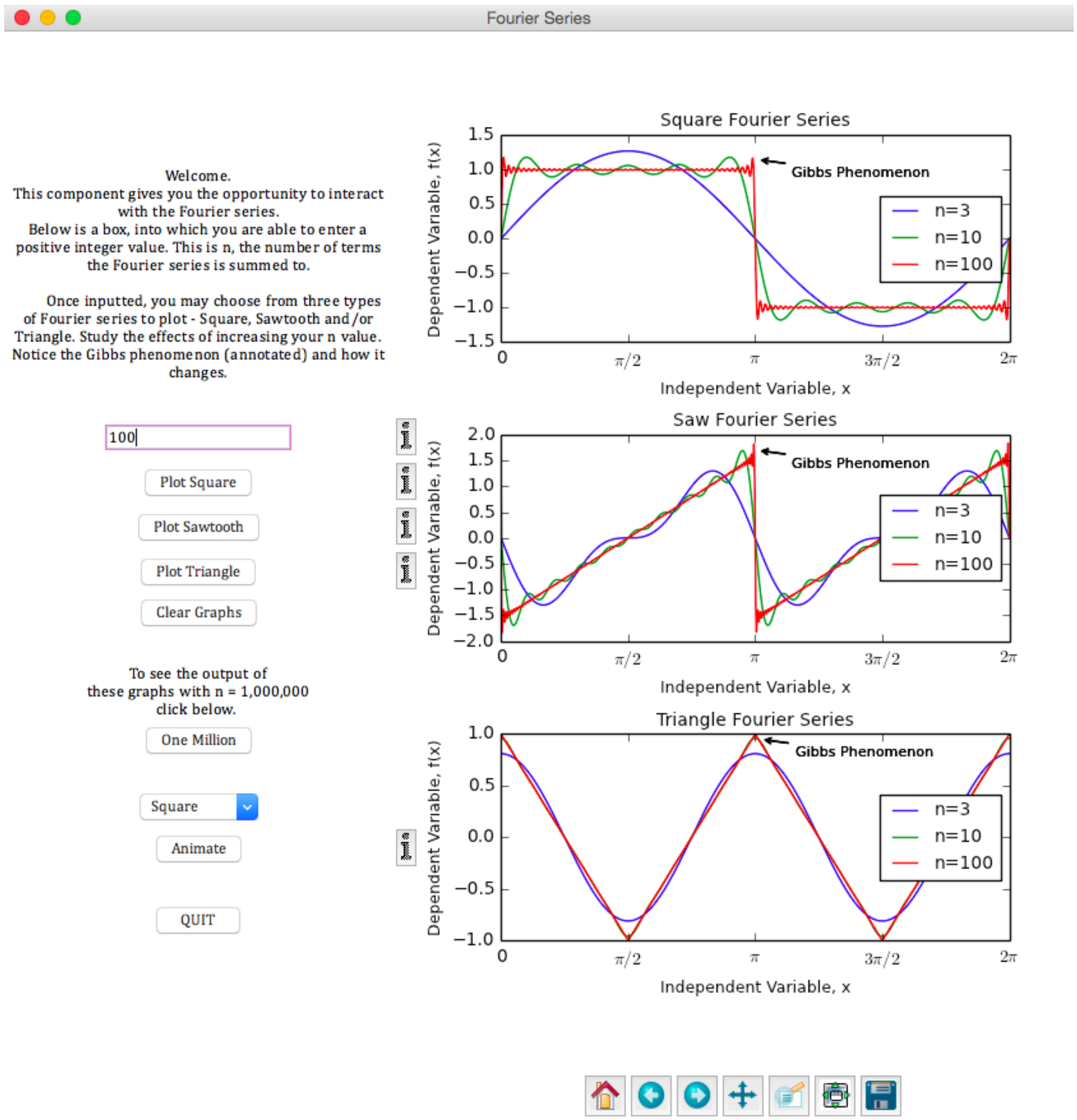

Figure 2: Screenshot displaying the main component of the GUI containing text, entry box, button, option menu and canvas widgets.

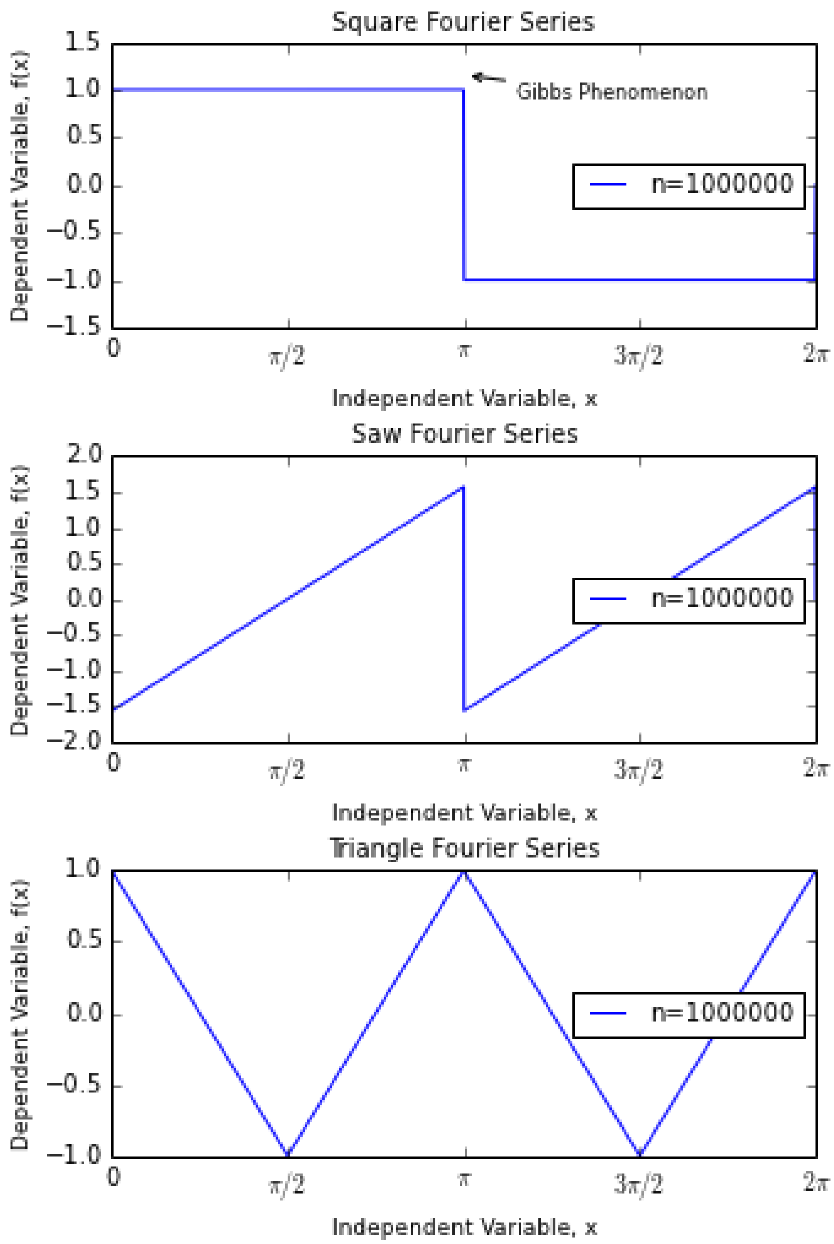

Figure 3: The output of the sum of the Fourier series to the millionth term.